UNIVERSIDADE DE SÃO PAULO
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Titles & Abstracts

Adriano C. Bezerra - IFGO (*adriano.bezerra@ifgoiano.edu.br*)
Rigidity and Stability Estimates for CMC-Submanifolds in $\mathbb{M}^{n+m}(c)$

In this talk we will establish conditions on the first eigenvalue of the stability operator and on the length of the traceless part of the second fundamental form of a complete submanifold M^n with constant mean curvature in $\mathbb{M}^{n+m}(c)$ to show that M^n has some geometric property of rigidity. This is a joint work with F. Manfio (ICMC-USP).

Benedito Leandro - UFG (*bleandroneto@ufg.br*)
Curve Shortening Flow on T^2

We present a characterization for the initial curve of a soliton solution for the curve shortening flow (CSF) on the torus of revolution. Furthermore, we describe the behavior of such curves by showing that the two ends of each curve are asymptotic to the equator.

Cleidinaldo A. Souza - UFPI (*aguiarnaldo@ufpi.edu.br*)
Hypersurfaces of cohomogeneity one into space forms

In this talk we present a complete classification of isometric immersions $f : M^n \to \mathbb{Q}_c^{n+1}$, for $n \geq 3$, where M^n is a compact Riemannian manifold of cohomogeneity one.

Ernani Ribeiro Jr - UFC (*ernani@mat.ufc.br*)
On the Hitchin-Thorpe inequality for 4-dimensional compact Ricci solitons

Ricci solitons are self-similar solutions of the Ricci flows and arise as the singularity models of the Ricci flows. In this talk, we discuss the geometry of 4-dimensional compact gradient Ricci solitons. It has been conjectured in 2006 by H.-D. Cao that every 4-dimensional compact Ricci soliton must satisfy the classical Hitchin-Thorpe inequality. We will show that such a conjecture is true under an upper bound condition on the range of the potential function. In addition, some volume estimates will also be discussed. This is a joint work with D. Zhou and X. Cheng.

Fábio Reis dos Santos - UFPE (*fabio.reis@ufpe.br*)
On complete submanifolds with parallel normalized mean curvature in productspaces

A Simons type formula for submanifolds with parallel normalized mean curvature vector field (pnmc submanifolds) in the product spaces $M^n(c) \times \mathbb{R}$, where $M^n(c)$ is a space form with constant sectional curvature $c \in \{-1, 1\}$ it will be shown. As an application, it will obtained rigidity results for submanifolds with constant second mean curvature.

Florentiu Daniel Cibotaru - UFC (*daniel@mat.ufc.br*)
Kurdyka-Łojasiewicz functions and mapping cylinder neighborhoods

Kurdyka-Łojasiewicz functions are real-valued functions characterized by a differential inequality involving the norm of their gradient. The class of such functions is quite rich containing for example real analytic, distance (to submanifolds) or Morse functions. We will prove a topological property for the zero loci of such functions implying that they cannot be too pathological and give some motivation as to why a geometer should be interested in them. This is based on joint work with Fernando Galaz-García (Univ. of Durham).

Lino Grama - UNICAMP (*linograma@gmail.com*)
Projected Ricci flow and applications to flag manifolds

In this talk we will present a normalization for the homogeneous Ricci flow with natural compactness properties. Our method consists in appropriately normalizing the flow to a simplex and time reparametrizing it to get polynomial equations, obtaining what we call the *projected Ricci flow*. As an application, we present a detailed picture of the homogeneous Ricci flow for three isotropy-summands flag manifolds: phase portraits, basins of attractions, conjugation classes and collapsing phenomena. This is a joint work with R. M. Martins, M. Patrão, L. Seco, and L. D. Sperança.

Marcos Alexandrino - IME - USP (*malex@ime.usp.br*)
Singular Riemannian foliations, the semi local model and applications in analysis

A singular foliation on a Riemannian manifold M is called singular Riemannian foliation (SRF for short) if their leaves are locally equidistant, i.e., if each geodesic that is perpendicular to one leaf is perpendicular to every leaf it meets. A typical example is a partition of M into orbits of an isometric action. There are infinitely many examples of SRF with non homogenous leaves, among them the so called holonomy foliation, i.e., the natural singular foliation contained in each Euclidean fiber bundle with a connection compatible with the metric.

The aim of the talk is twofold. Firstly, we review the local model for any Singular Riemannian Foliation in a neighbourhood of a closed saturated submanifold of a regular stratum, based on a joint work with Marcelo K. Inagaki, Mateus de Melo, Ivan Struchiner.

Secondly, we briefly discuss how this model implies a Principle of Symmetric Criticality of Palais, that is used to prove the following result: if M admits an orbit like foliation (i.e, a closed SRF transversely infinitesimally homogenous), then there exists a metric with constant scalar curvature for which the foliation is still an orbit like foliation (i.e, a foliated Yamabe's problem). This second part is based on a joint work with Leonardo Cavenaghi.

We hope to present a talk for a general public of geometers with no previous knowledge on foliations.

Marcus Marrocos - UFAM (*marcusmarrocos@gmail.com*)
Triviality and nonexistence results for gradient Ricci soliton

A complete Riemannian metric g on a smooth manifold M is a gradient Ricci soliton if there exists a smooth function ϕ on M such that the Ricci tensor of g is given by

$$Ric + \nabla^2\phi = \lambda g,$$

for some constant λ. Also gradient Ricci solitons are self-similar solutions of the Ricci flow, and often arise as possible singularity models of this flow. Our goal is to study gradient Ricci solitons that are (or not) realized as warped metrics g on $M = B^n \times F^m$. Albeit the class of warped metrics with nonconstant warping functions provides a rich class of examples in Riemannian geometry, in this work, under natural geometric assumptions on the warping function as well as on the potential function of a gradient Ricci soliton warped metric, we give some obstructions for constructing such a metric. This is a joint work with José N. V. Gomes (UFSCar) and Andrian V.C. Ribeiro (UEA).

Maria Fernanda Elbert - UFRJ (*fernanda@im.ufrj.br*)
Estabilidade de H_r-hipersuperfícies

A curvatura média de ordem r, H_r, é definida como a r-ésima função simétrica da curvaturas principais. Chamaremos de H_r-hipersuperfíces, as hipersuperfícies com H_r constante. Estamos interessados no estudo de estabilidade desses objetos, buscando generalizar o estudo clássico da estabilidade das hipersuperfícies de curvatura média constante (caso $r = 1$). Cabe ressaltar que para $r = 1$, o estudo da estabilidade, iniciado há mais de 40 anos, está bem estabelecido, tanto para o caso de *bordo fixo*, quanto para hipersuperfícies *de bordo livre*.

No final da década de 90, a estabilidade para o caso $r > 1$ teve importante avanço com o trabalho de L.Barbosa e G. Colares, onde os autores trataram o caso de hipersuperfícies imersas em formas espaciais e com bordo fixo. Alguns entraves técnicos impediram avanços desde então. Nessa palestra, pretendo introduzir uma nova abordagem ao estudo de estabilidade das H_r-hipersuperfícies. Para o caso de bordo fixo, falarei de resultados de um artigo recente com Barbara Nelli. Para o caso de bordo livre, apresentarei resultados de um artigo em fase de preparação com Leonardo Damasceno.

Matheus Vieira - UFES (*mathbhv@gmail.com*)
Biharmonic hypersurfaces in hemispheres

In this paper we consider the Balmus-Montaldo-Oniciuc conjecture in the case of hemispheres. We prove that a compact non-minimal biharmonic hypersurface in a hemisphere must be the small hypersphere, provided that $n^2 - H^2$ does not change sign.

Miguel I. Jimenez - ICMC - USP (*mathbhv@gmail.com*)
Infinitesimally Bonnet bendable hypersurfaces

The classical Bonnet problem deals with those surfaces in $\mathbb{R}^3$ that are not determined, up to rigid motion, by their induced metric and mean curvature function. In other words, it deals with surfaces in $\mathbb{R}^3$ that admit isometric deformations sharing the same mean curvature function. A higher dimensional version of this problem, for hypersurfaces of the Euclidean space, was considered by Kokubu in 1992.

In this talk we present the infinitesimal version of Kokubu's work. We are interested in those hypersurfaces that admit smooth variations by immersions whose metrics and mean curvature functions are preserved "up to the first order". That is, we classify the hypersurfaces $f\colon M^n \to \mathbb{R}^{n+1}$ that admit non-trivial variations by immersions $f_t\colon M^n \to \mathbb{R}^{n+1}$ whose induced metrics g_t and mean curvature functions H_t satisfy

$$\frac{\partial}{\partial t}|_{t=0} g_t = 0 = \frac{\partial}{\partial t}|_{t=0} H_t.$$

This is a joint work with Ruy Tojeiro.

Ronaldo F. Lima - UFRN (*ronaldo.freire@ufrn.br*)
Weingarten Flows in Riemannian Manifolds

In this talk, we consider flows $F_t : M^n \to \overline{M}^{n+1}$, called *Weingarten flows*, in which the hypersurface $F_t(M)$ evolves in the direction of its normal vector with speed given by a monotone increasing, symmetric and homogeneous function of its principal curvatures. We obtain existence results in the case the hypersurfaces F_t are all isoparametric and $\overline{M}$ is either a simply connected space form or a rank-one symmetric space of noncompact type. Finally, by means of a result by R. Hamilton, we establish an avoidance principle for Weingarten flows in totally convex Riemannian manifolds.

Rondinelle M. Batista - UFPI (*rmarcolino@ufpi.edu.br*)
Charged Hawking mass and local rigidity of minimal two-spheres in three-manifolds

In this talk we discuss the rigidity of minimal two-spheres Σ that locally maximize the charged Hawking mass on a Riemannian three-manifold with suitable lower bound on its scalar curvature. Assuming strict stability of Σ, we prove that a neighborhood of it in M^3 is isometric to one of the deSitter Reissner-Nordstrom. This is a joint work with Halyson Baltazar (UFPI) and Abdênago Barros (UFC).

Ruy Tojeiro - ICMC - USP (*tojeiro@icmc.usp.br*)
Moebius deformable hypersurfaces

Li, Ma and Wang investigated in [*Deformations of hypersurfaces preserving the Möbius metric and a reduction theorem*, Adv. Math. 256 (2014), 156-205] the interesting class of Moebius deformable hypersurfaces, that is, the umbilic-free Euclidean hypersurfaces $f\colon M^n \to \mathbb{R}^{n+1}$ that admit non-trivial deformations preserving the Moebius metric. The classification of Moebius deformable hypersurfaces of dimension $n \geq 4$ stated in the aforementioned article, however, misses a large class of examples. In this talk I will report on a joint work with M. I. Jimenez, in which we first complete that classification for $n \geq 5$. Then we introduce the notion of an infinitesimal Moebius variation of an umbilic-free immersion $f\colon M^n \to \mathbb{R}^m$ into Euclidean space as a one-parameter family of immersions $f_t\colon M^n \to \mathbb{R}^m$, with $t \in (-\epsilon, \epsilon)$ and $f_0 = f$, such that the Moebius metrics determined by f_t coincide *up to the first order*. We characterize isometric immersions $f\colon M^n \to \mathbb{R}^m$ of arbitrary codimension that admit a non-trivial infinitesimal Moebius variation among those that admit a non-trivial conformal infinitesimal bending, and use such characterization to classify the larger class of umbilic-free Euclidean hypersurfaces of dimension $n \geq 5$ that admit non-trivial infinitesimal Moebius variations. Based on joint work with M.I. Jimenez.

Valter Borges - UFPA (*valterborges@ufpa.br*)

Hamilton type inequalities and classification of ρ-Einstein solitons

In Theorem 20.1 of [4], Hamilton proved an identity for steady Ricci solitons relating the scalar curvature to the first derivative of the potential function. The method used in a general Ricci soliton tells that $R + |\nabla f|^2 - 2\lambda f$ is constant. This is called by some authors the Hamilton identity. When used in combination with other identities, it returns important analytic and geometrical information about Ricci solitons.

For $\rho \in \mathbb{R}$, a ρ-Einstein soliton is a Riemannian manifold where the equation

$$Ric + \nabla\nabla f = (\rho R + \lambda)g$$

is satisfied for a constant λ. This was first considered by Catino and Mazzieri in [2]. Despite being a perturbation of Ricci solitons, case corresponding to $\rho = 0$, it is not known whether a Hamilton type identity holds for these manifolds.

In this talk, we exhibit substitutes for the Hamilton identity on ρ-Einstein solitons which were obtained under some restrictions in [2] when $\rho \in (0, 1/2(n-1))$ and $n \geq 3$, in [3] when $\rho = 1/4$ and $n = 3$, and in [1] when $\rho = 1/2(n-1)$, where in this last one there is no assumptions. We also explore some of their geometric consequences.

[1] Borges, V. *On complete gradient Schouten solitons.* Nonlinear Analysis 221, 112883, (2022).

[2] Catino, G., Mazzieri, L. *Gradient Einstein solitons.* Nonlinear Analysis 132, 66-94 (2016).

[3] Catino, G., Mazzieri, L., Mongodi, S. *Rigidity of gradient Einstein shrinkers.* Communications in Contemporary Mathematics, 17(06), 1550046, (2015).

[4] Hamilton, R. *The formations of singularities in the Ricci Flow.* Surveys in differential geometry 2, no. 1, 7-136 (1993).

Vinicius Ramos - IMPA (*vgbramos@impa.br*)

A conjectura de Viterbo e sistemas integráveis

Viterbo conjecturou a existência de uma desigualdade relacionando capacidades simpléticas e o volume. Isso pode ser visto como uma generalização de uma desigualdade sistólica.

Nessa palestra, falarei um pouco dessa história e como a teoria de sistemas integráveis gera domínios em que a conjectura de Viterbo pode ser verificada. Falarei de trabalhos em conjunto com Gutt-Hutchings e Ostrover-Sepe.

Yunelsy N. Alvarez - IME - USP (*ynapolez@gmail.com*)

Solvability criteria for Dirichlet problems of mean curvature type in Riemannian manifolds

In this talk, we investigate the existence of graphs with prescribed mean curvature in Riemannian manifolds. Specifically, we show that a condition -inherited from the Euclidean setting- is sufficient for the solvability of the Dirichlet problem for prescribed mean curvature equations in a large class of manifolds.
